LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA

04

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INTRODUCTION

A major advantage of analyzing circuit using Kirchhoff's laws is that, original circuit configuration is not tampered.

A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved.

To handle the complexity, engineers have developed some theorems to simplify the circuit analysis.

These theorems include, superposition theorem, source transformation, Thevenin's theorem, Norton's theorem and maximum power transfer theorem.

LINEARITY PROPERTY

Linearity is the property of an element describing a linear relationship between cause and effect.

The property is a combination of both the homogeneity property and additivity property.

The Homogeneity property requires that if the input is multiplied by a constant, then the output is also multiplied by the same constant.

For example, Ohm's law;

$$v = iR$$

LINEARITY PROPERTY

If the current is increased by a constant k, then the voltage increases correspondingly by k.

$$kiR = kv$$

The Additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

For example, application of Ohm's law on two resistors;

$$v_1 = i_1R$$

$$v_2 = i_2R$$

LINEARITY PROPERTY

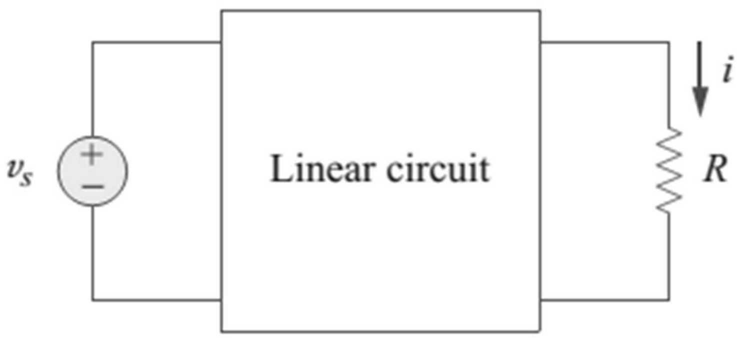
According to additivity property;

$$v = (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2$$

Here resistor is a linear circuit element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

LINEAR CIRCUIT

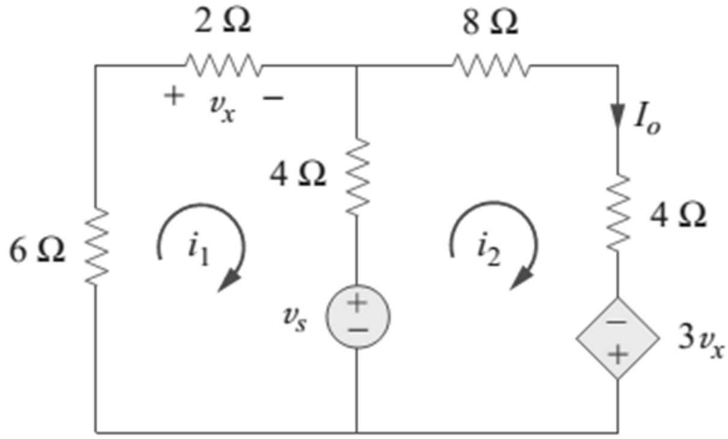
A Linear Circuit is one whose output is linearly related (directly proportional) to its input.



Throughout the circuit analysis, we only consider the linear circuits.

PROBLEMS

Find the current I_o, when v_s=12 V and v_s=24 V?



(12/76 A, 24/76 A)

SUPERPOSITION THEOREM

The Superposition Theorem states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or current through) that element due to each independent source acting alone.

The superposition theorem helps to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

One independent source at a time is considered while all other independent sources are turned off.

SUPERPOSITION THEOREM

An independent voltage source is replaced by a $0\ V$ or short circuit element.

An independent current source is replaced by $0\ A$ or open circuit.

Dependent sources are left intact because they are controlled by circuit variables.

Analyzing circuit using superposition, involves more work.

This reduces the complexity of circuit.

SUPERPOSITION THEOREM

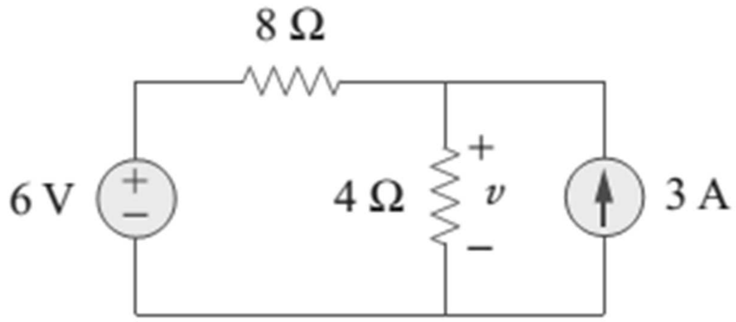
Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

The superposition theorem is based on the linearity property.

PROBLEMS

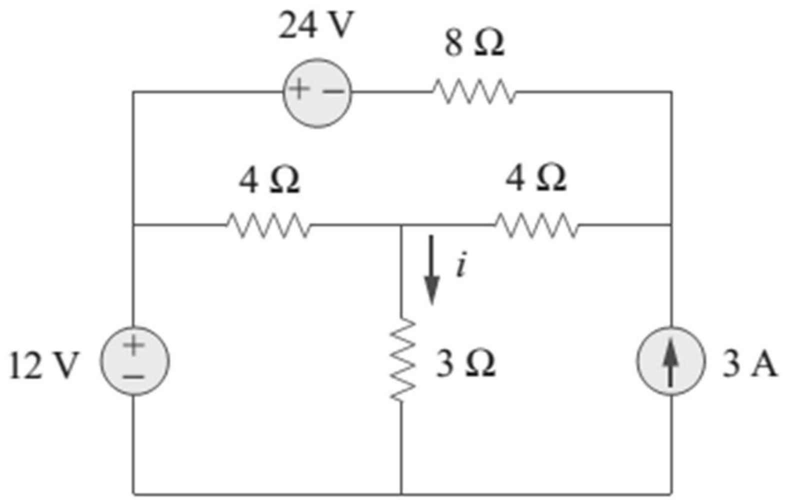
Find the voltage?



(2+8=10 V)

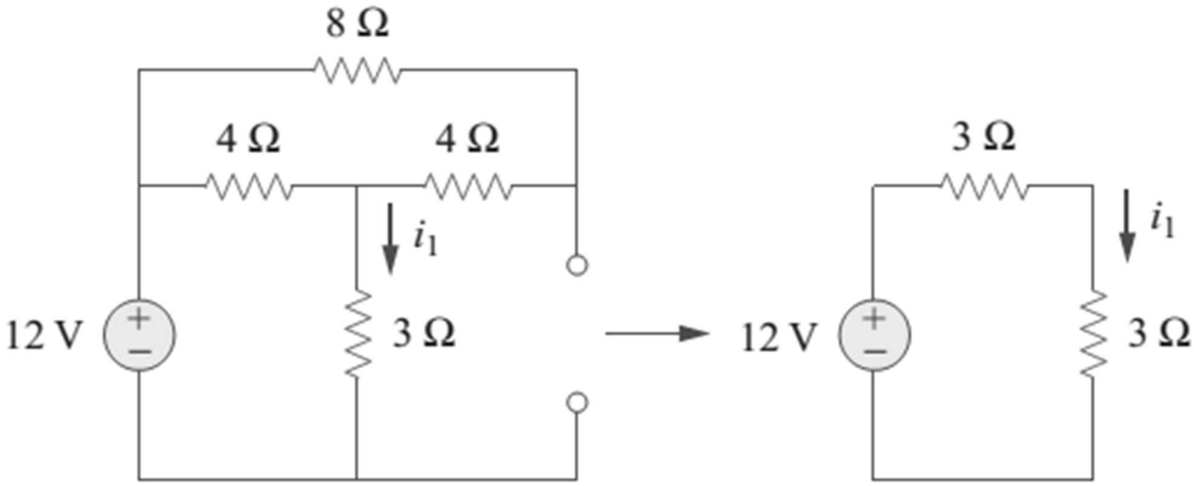
PROBLEMS

Find the current?



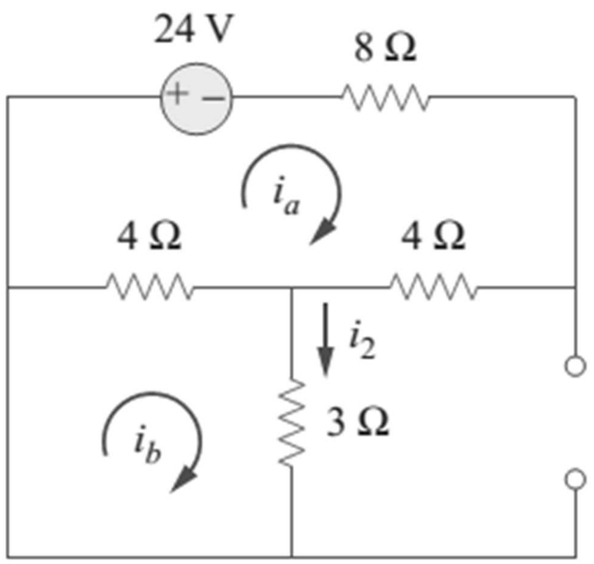
(2-1+1=2 A)

PROBLEMS



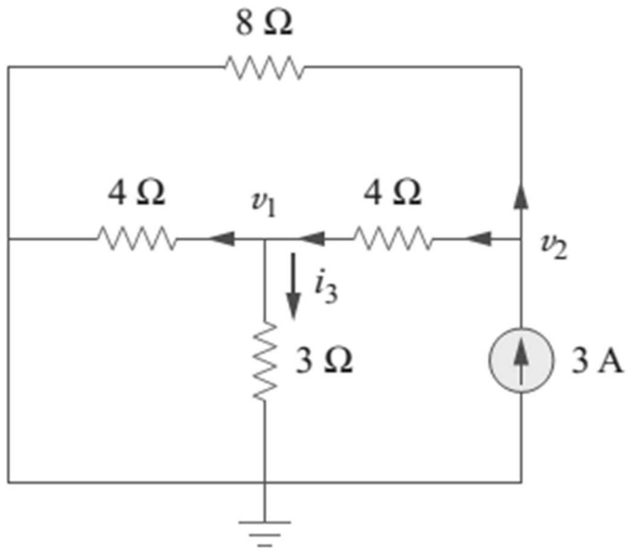
(2 A)

PROBLEMS



(-1 A)

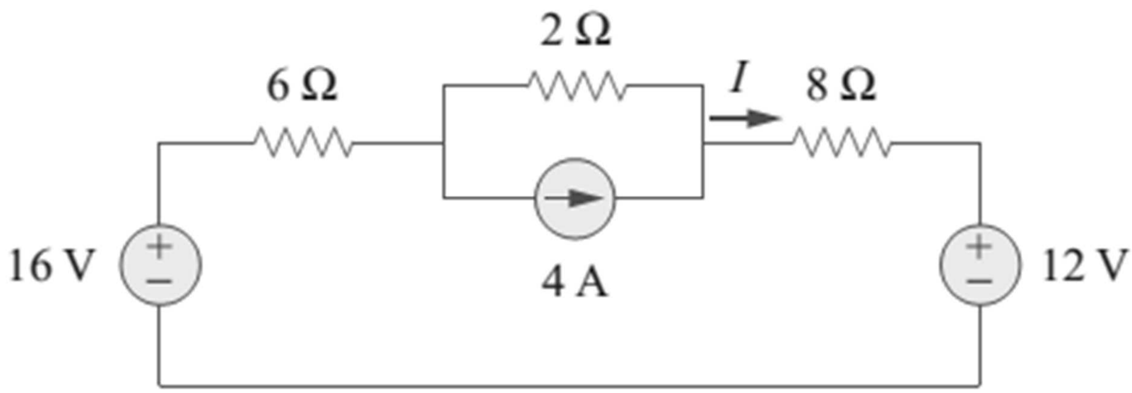
PROBLEMS



(1 A)

PROBLEMS

Find the current?

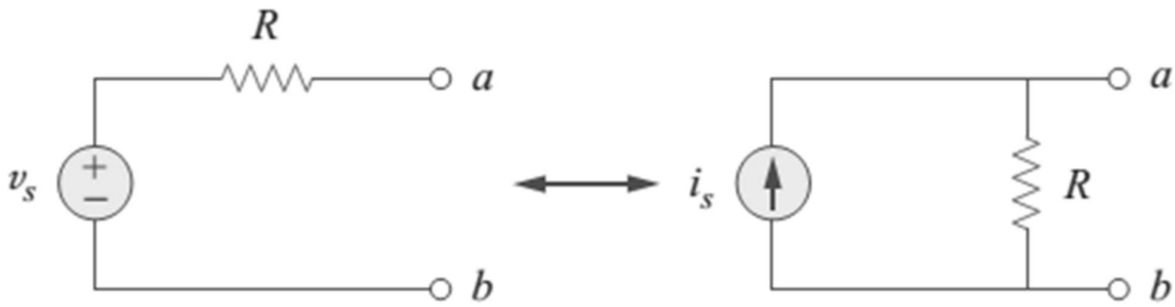


(1-0.75+0.5=0.75 A)

SOURCE TRANSFORMATION

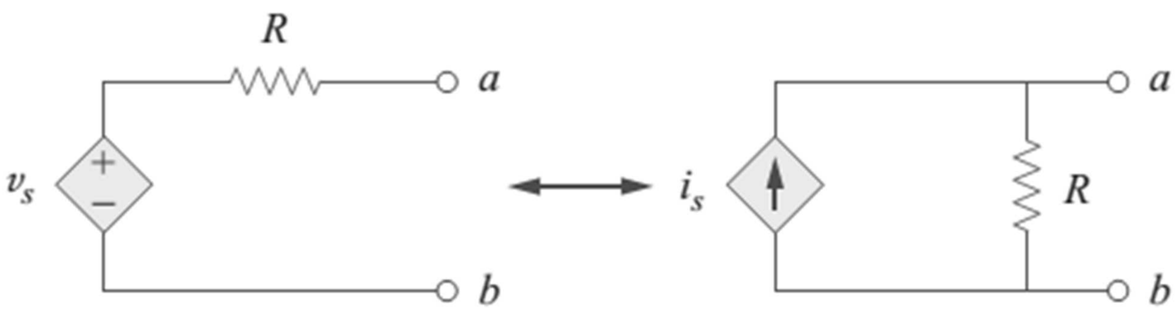
A Source Transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

Transformation of independent sources;



SOURCE TRANSFORMATION

Transformation of dependent sources;



Ohm's law is applied to find unknown voltage or current.

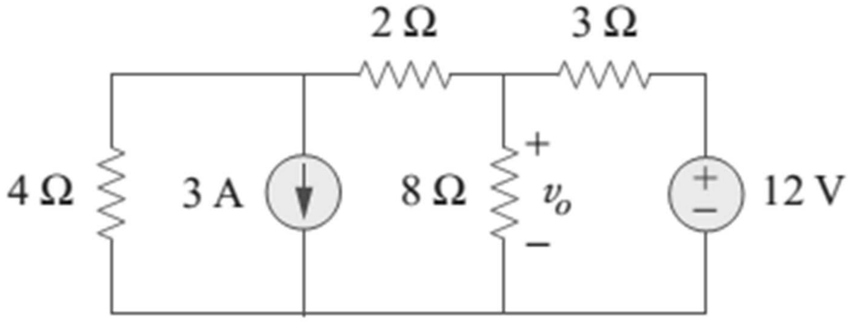
$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

SOURCE TRANSFORMATION

The arrow head of the current source is directed towards the positive terminal of the voltage source.

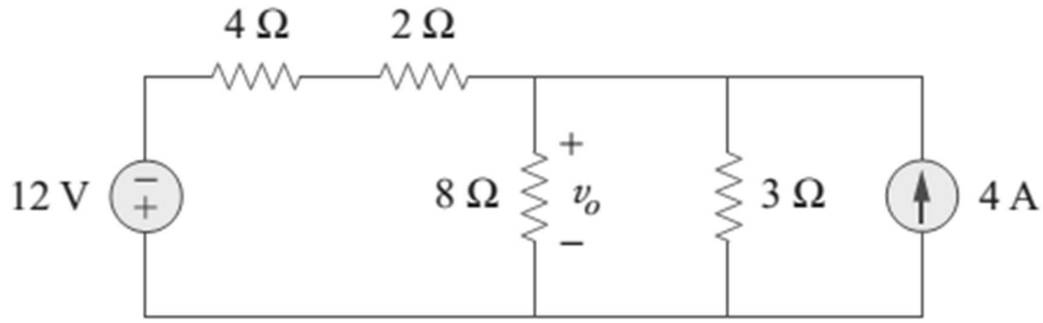
PROBLEMS

Find the voltage?

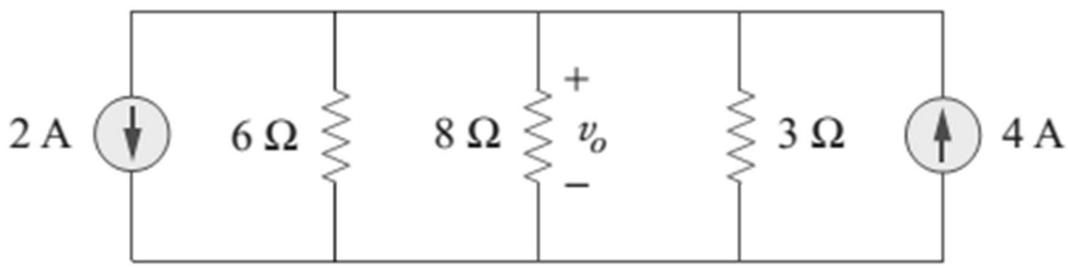


(3.2 V)

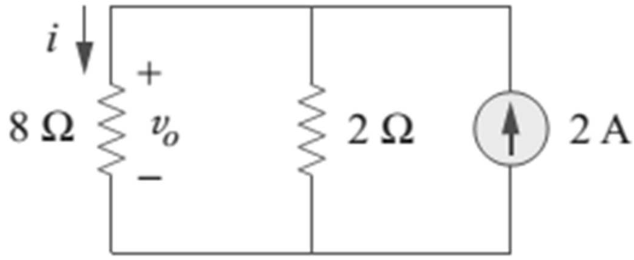
PROBLEMS



PROBLEMS

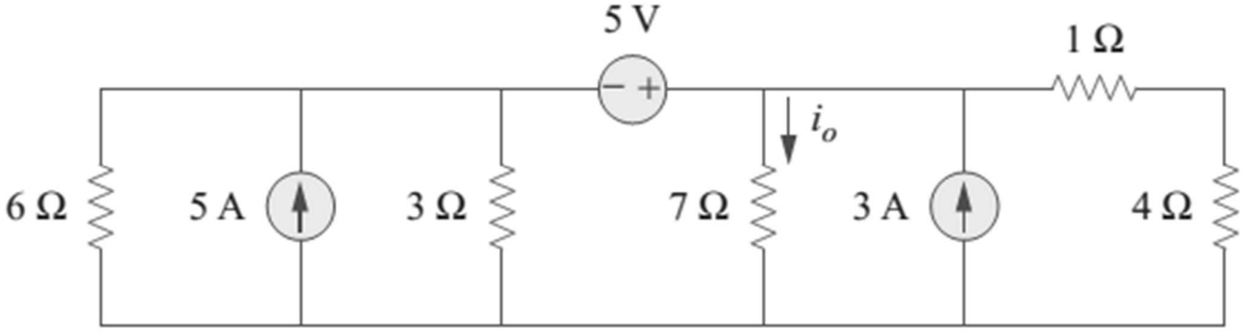


PROBLEMS



PROBLEMS

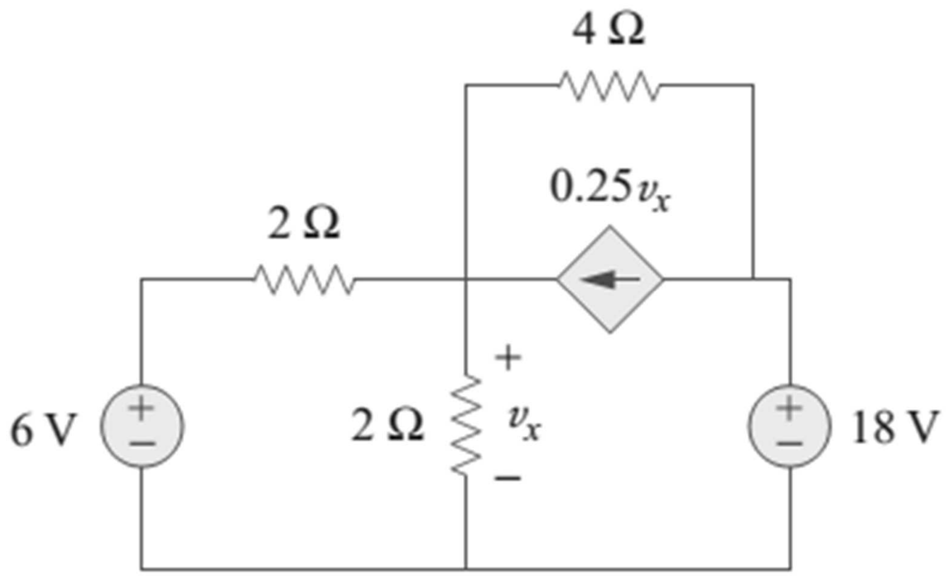
Find the current?



(1.78 A)

PROBLEMS

Find the voltage?



(7.5 V)

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 04 – Circuit Theorems (4.1 – 4.4)

Exercise Problems: 4.1 – 4.32

Do exercise problem yourself.

THEVENIN'S THEOREM

It often occurs in practice that a particular element in a circuit is variable while other elements are fixed.

For example, a household outlet terminal may be connected to different appliances constituting a variable load.

Each time the variable element is changed, the entire circuit has to be analyzed all over again.

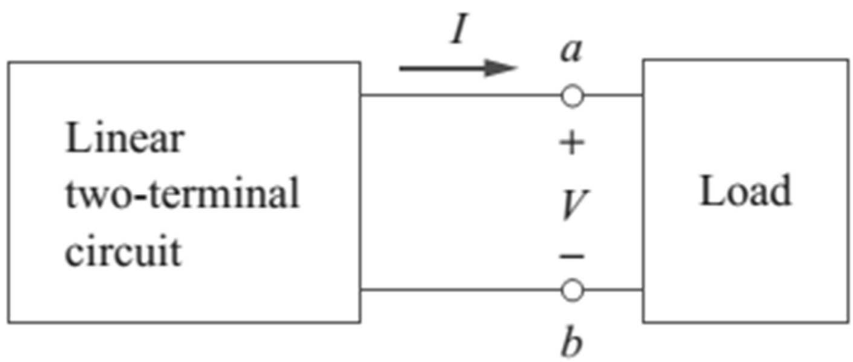
To avoid this problem, Thevenin's theorem provides the technique by which fixed part of the circuit is replaced by an equivalent circuit.

THEVENIN'S THEOREM

Thevenin's Theorem states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open circuit voltage at the terminals and R_{th} is the input or equivalent resistance at the terminals when independent sources are turned off.

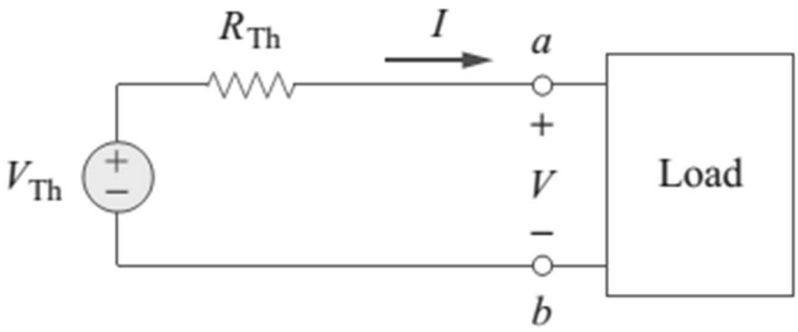
Original

Circuit;



THEVENIN'S THEOREM

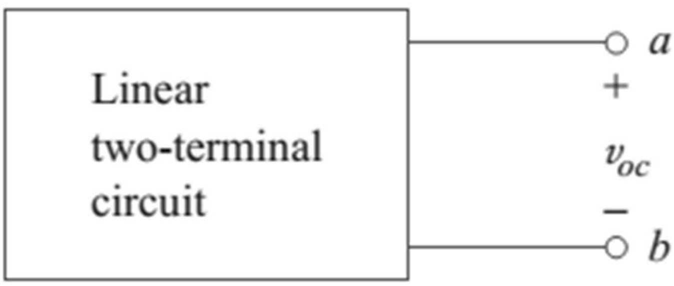
Thevenin's equivalent circuit;



THEVENIN'S THEOREM

Thus V_{th} is the open circuit voltage as shown;

$$V_{\mathrm{Th}} = v_{oc}$$

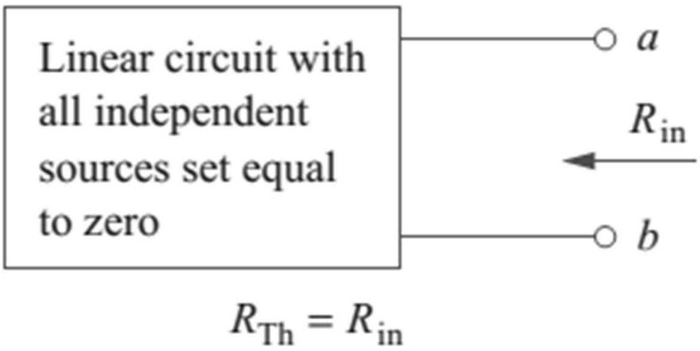


$V_{\mathrm{Th}} = v_{oc}$

THEVENIN'S THEOREM

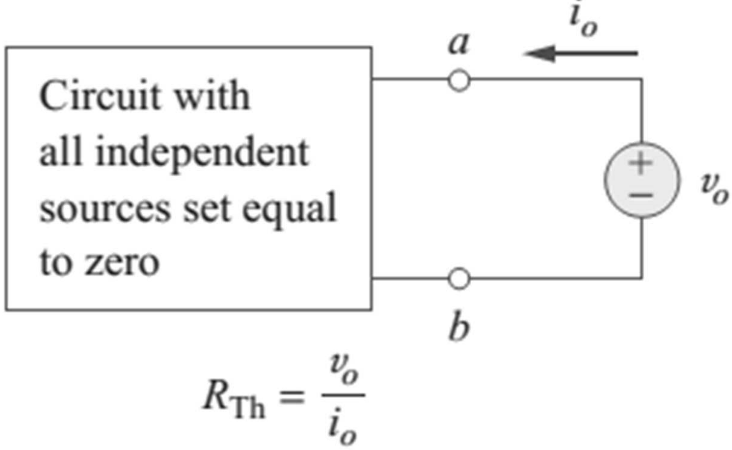
There are two cases to determine R_{th};

Case 1: If the network has no dependent sources, turn off all independent sources and calculate equivalent resistance looking between terminals a and b.



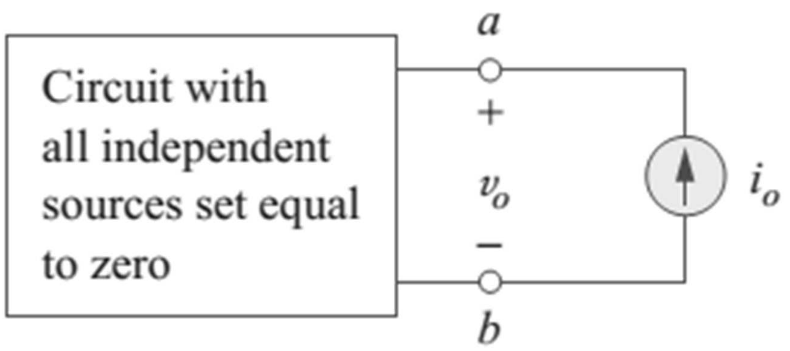
THEVENIN'S THEOREM

Case 2: If the network has dependent sources, turn off all independent sources and apply voltage source v_o (or current source i_o) between terminals a and b. Last, calculate the value of R_{th}.



THEVENIN'S THEOREM

Case 2: If the network has dependent sources, turn off all independent sources and apply voltage source v_o (or current source i_o) between terminals a and b. Last, calculate the value of R_{th}.



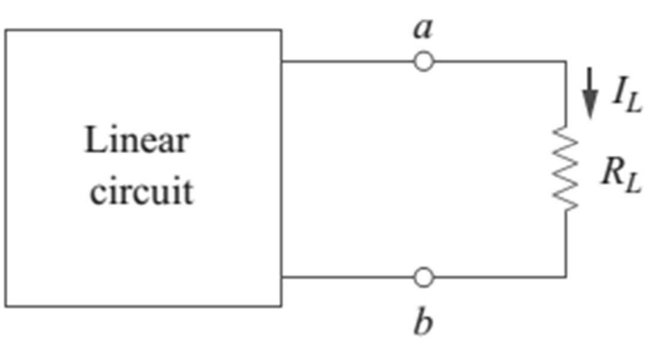
$$R_{\text{Th}} = \frac{v_o}{i_o}$$

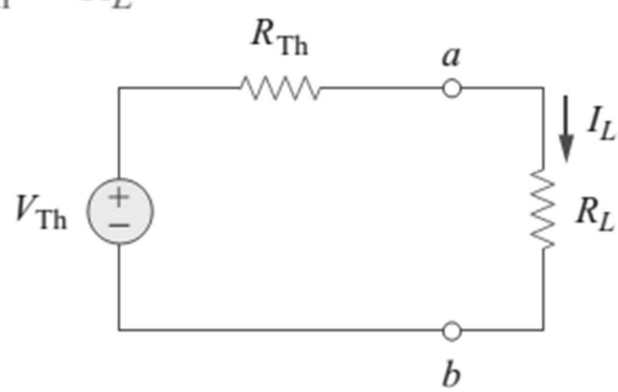
THEVENIN'S THEOREM

The load current and voltage can be calculated as;

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

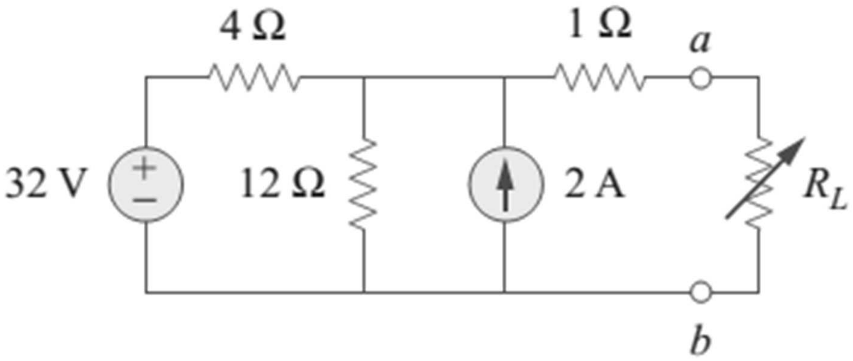
$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$





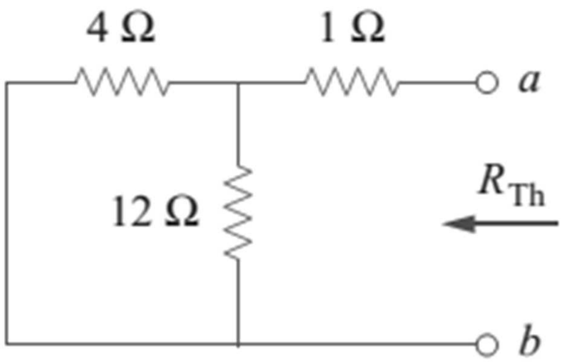
PROBLEMS

Find the current, if R_L=6, 16, 36 ohm?



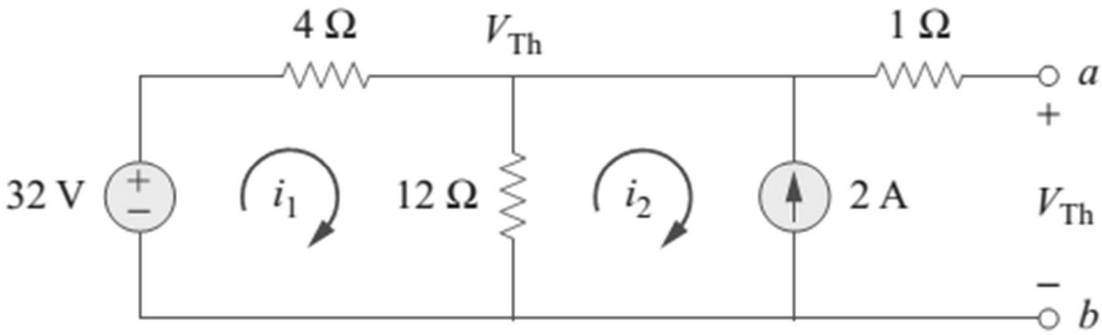
(3, 1.5, 0.75 A)

PROBLEMS



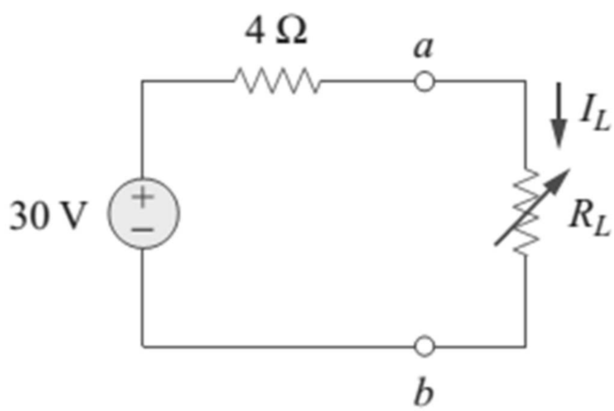
(4 ohm)

PROBLEMS



(30 V)

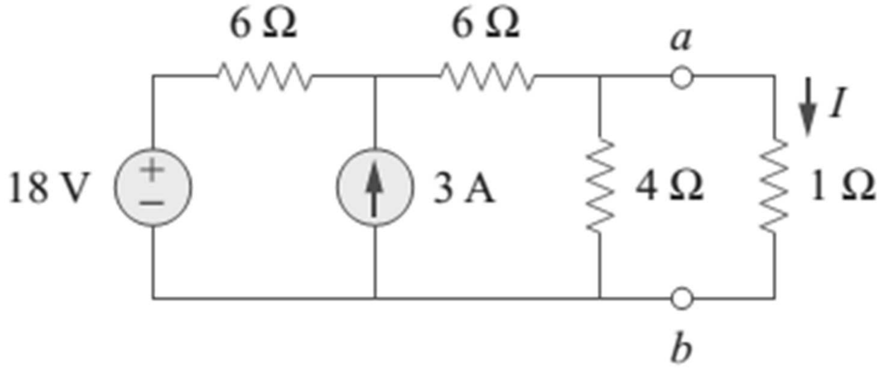
PROBLEMS



(3, 1.5, 0.75 A)

PROBLEMS

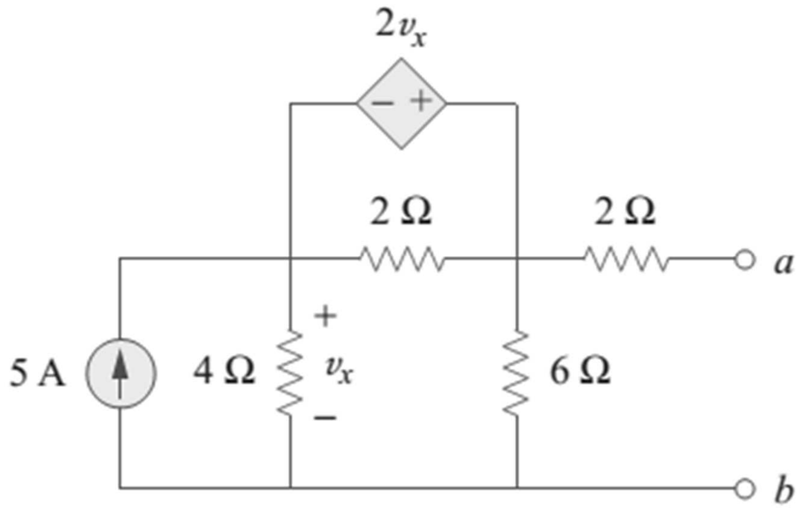
Find the current?



(9 V, 3 ohm, 2.25 A)

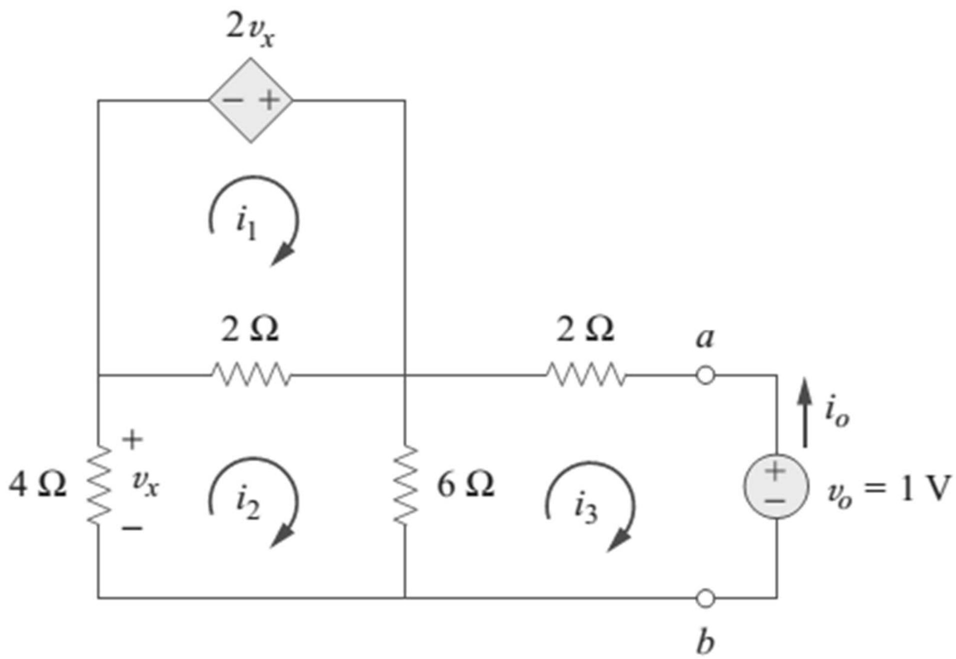
PROBLEMS

Find the Thevenin's equivalent network?

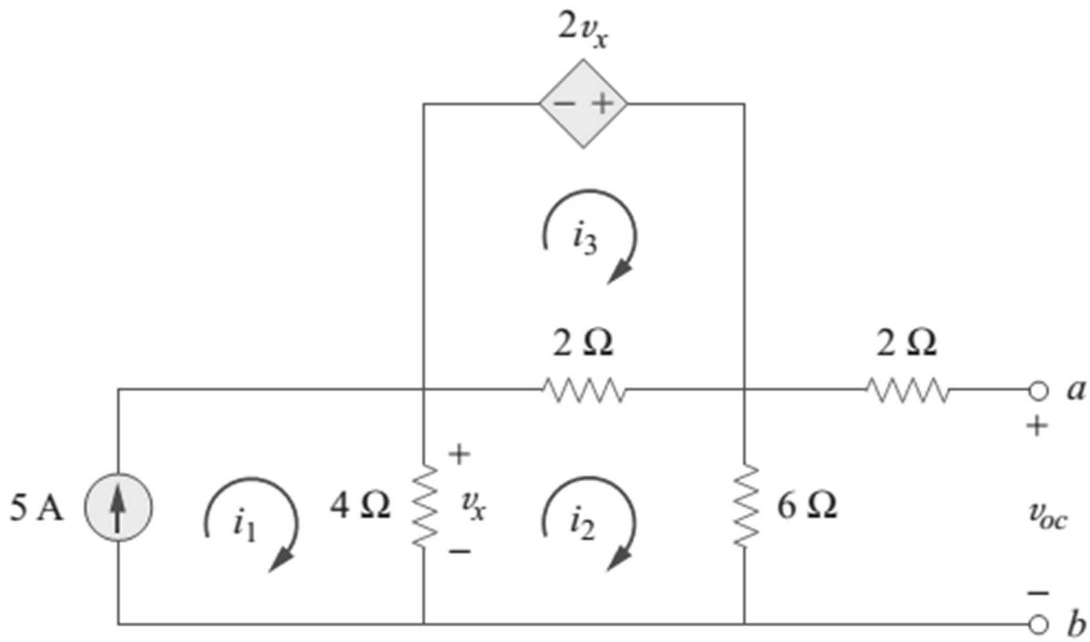


(20 V, 6 ohm)

PROBLEMS



PROBLEMS

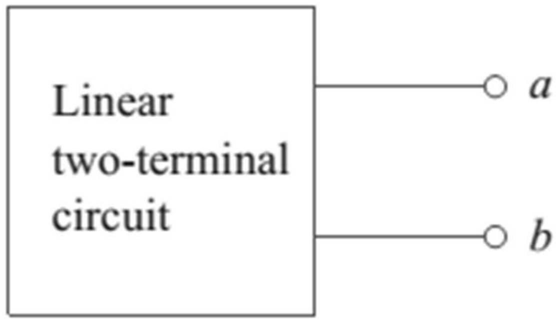


NORTON'S THEOREM

Thevenin's Theorem states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when independent sources are turned off.

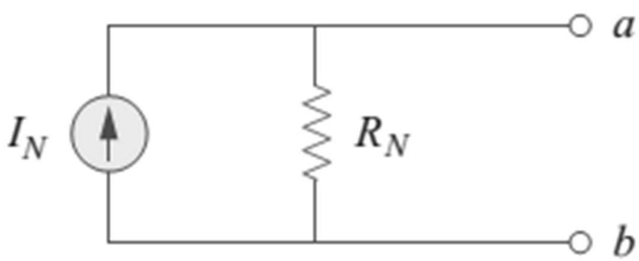
Original

Circuit;



NORTON'S THEOREM

Norton's equivalent circuit;



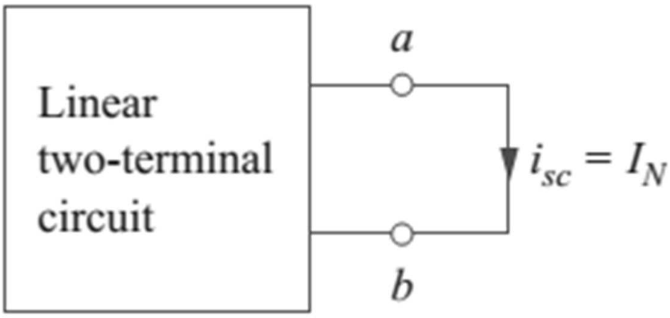
The equivalent resistance in case of both Norton and Thevenin are equal.

$$R_N = R_{Th}$$

NORTON'S THEOREM

The short circuit current through terminals a and b will be equal to Norton's current.

$$I_N = i_{sc}$$



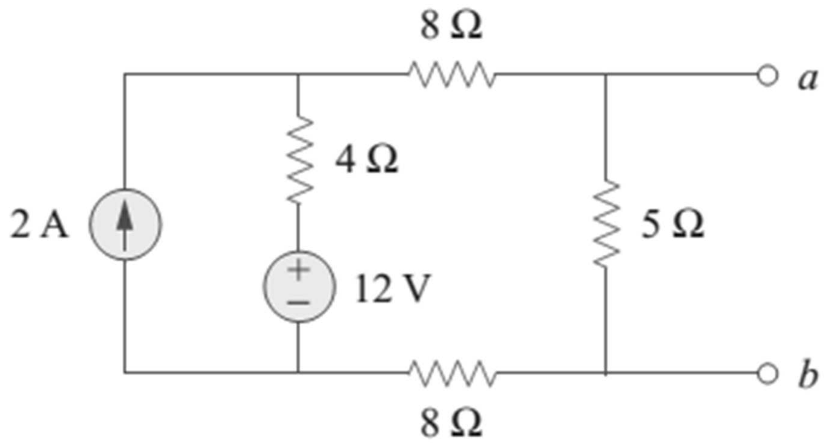
NORTON'S THEOREM

The parameters of Thevenin's and Norton's equivalent circuits may be expressed as;

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$

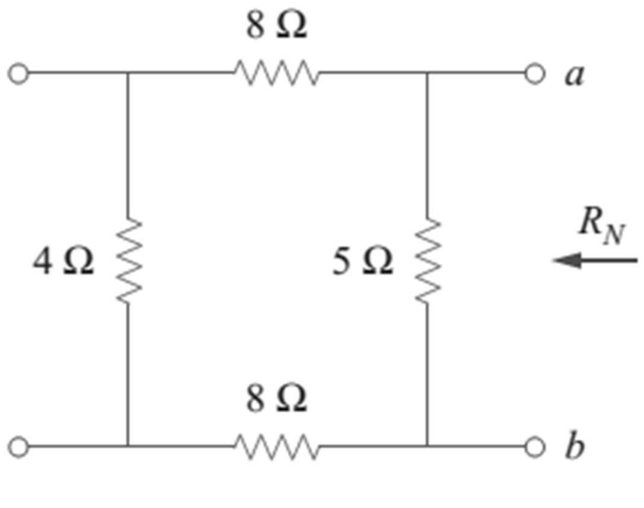
PROBLEMS

Find Norton's equivalent circuit?

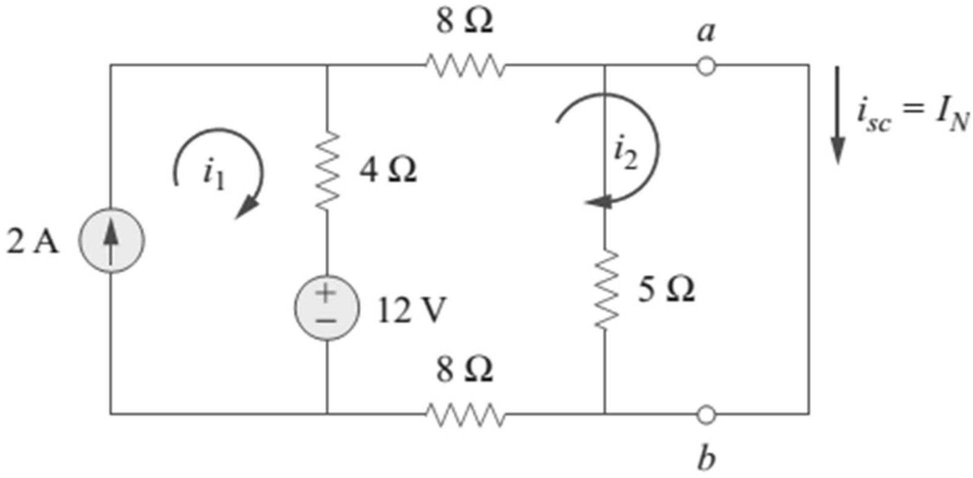


(4 ohm, 1 A)

PROBLEMS

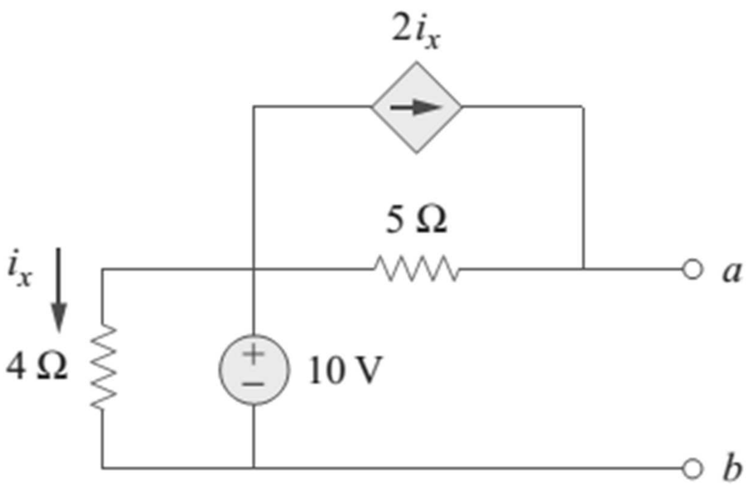


PROBLEMS



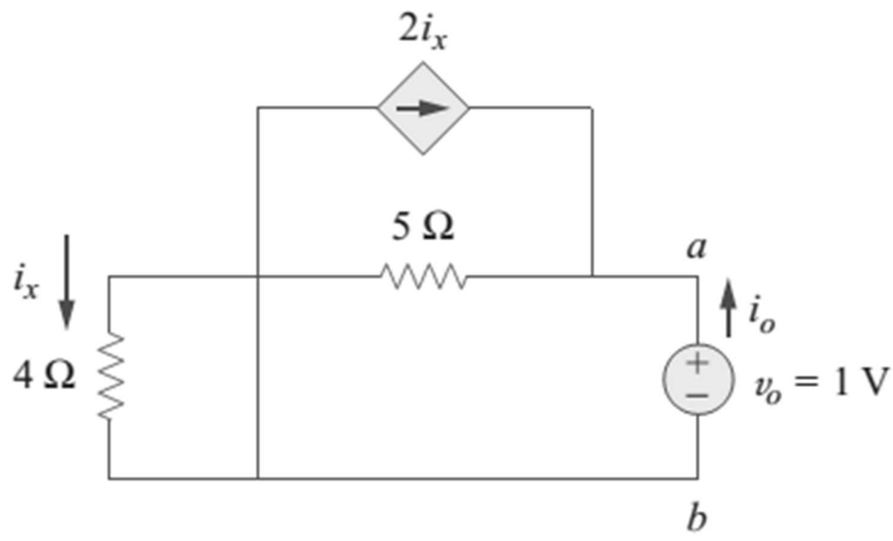
PROBLEMS

Find Norton's equivalent circuit?

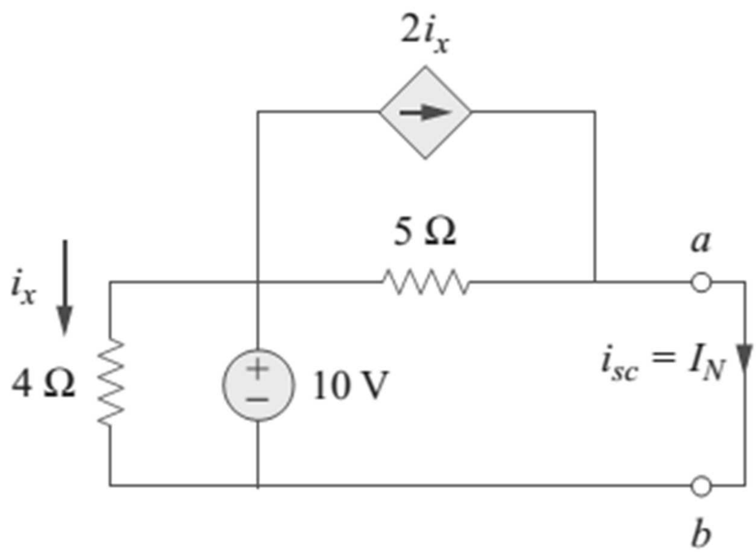


(5 ohm, 7 A)

PROBLEMS



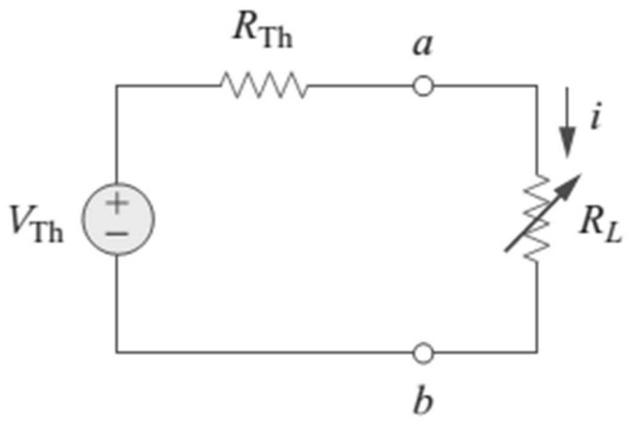
PROBLEMS



MAXIMUM POWER TRANSFER THEOREM

In many practical situations, a circuit is designed to provide the maximum power to the load.

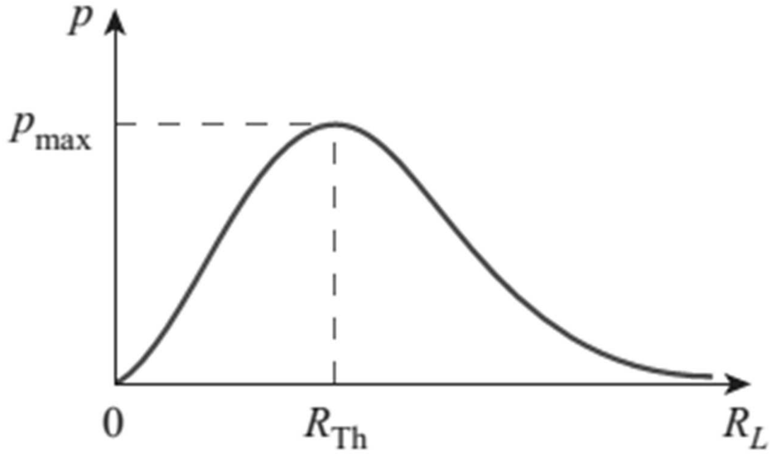
Thevenin's equivalent is useful in finding the maximum power a linear circuit can deliver to the load.



MAXIMUM POWER TRANSFER THEOREM

The power delivered to the load may be expressed as;

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



MAXIMUM POWER TRANSFER THEOREM

Maximum Power is transferred to the load when the load resistance equals the Thevenin's resistance as seen from the load *(R_L=R_{Th})*.

Maximizing the power equation of the load as;

$$p = i^2 R_L = \left(\frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}\right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4}\right]$$

$$= V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3}\right] = 0$$

MAXIMUM POWER TRANSFER THEOREM

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L)$$

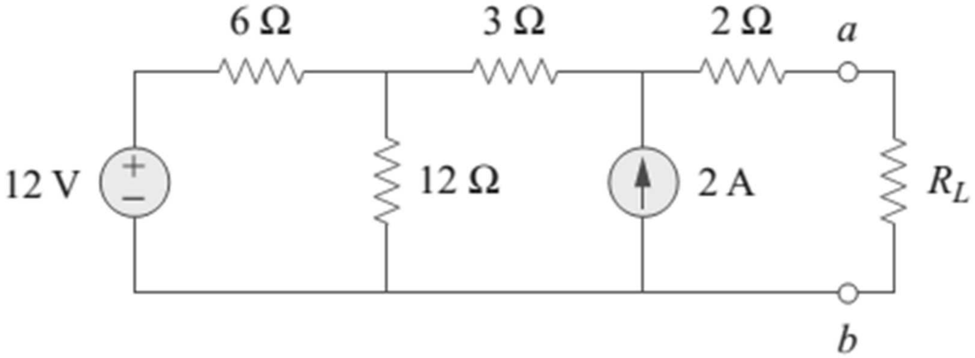
$$R_L = R_{\text{Th}}$$

Thus maximum power transferred to the load;

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

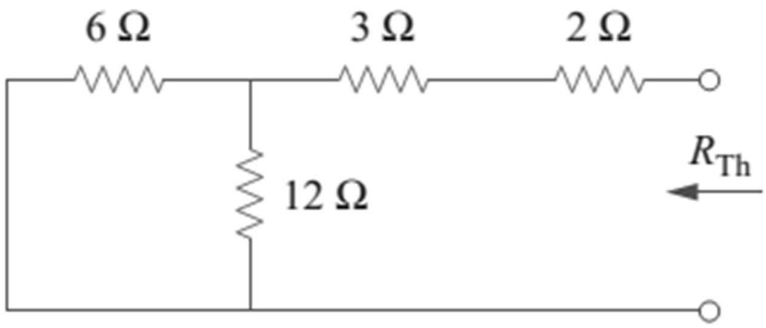
PROBLEMS

Find value of R_L for maximum power transfer?

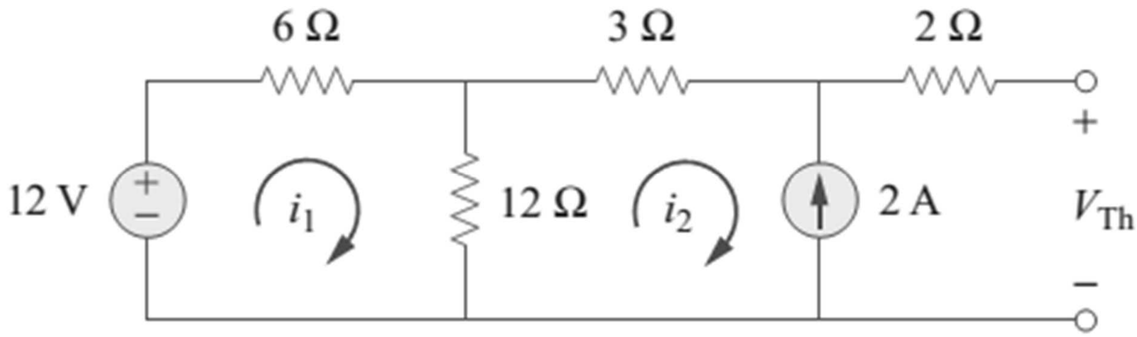


(9 ohm, 22 V, 13.44 W)

PROBLEMS



PROBLEMS



REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 04 – Circuit Theorems (4.5 – 4.8)

Exercise Problems: 4.33 – 4.92

Do exercise problem yourself.